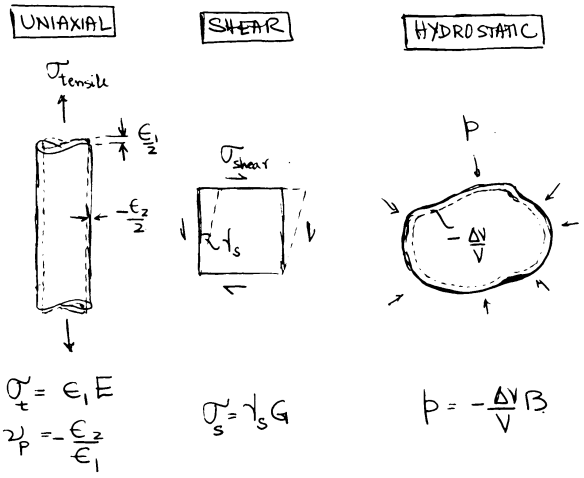
#5: The elastic constants



Shear: deformation at constant volume: shape changes but the volume remains the same.

Hydrostatic: deformation at constant shape.

Tensile or Young's Modulus $E = \frac{\sigma_{uni}}{\varepsilon_{uni}}$

Shear Modulus G

Bulk Modulus B

Poisson's Ratio $\nu = -\frac{\varepsilon_{uni}}{\varepsilon_{trans}}$

In an isotropic material only two of these are independent for example

(E, ν)

(E, G)

(E, B)

They are related as follows,

$$G = \frac{E}{2(1+\nu)}$$

$$B = \frac{2G(1+\nu)}{3(1-2\nu)}$$

$$B = \frac{E}{3(1-2\nu)}$$

Note that the Bulk modulus goes to infinity if $\nu \to 0.5$ incompressible deformation.

In a uniaxial test we measure: $\sigma_1, \varepsilon_1, \varepsilon_2$ ($\varepsilon_3 = \varepsilon_2$)

$$\ell n \frac{L_1 L_2 L_3}{L_1^0 L_2^0 L_3^0} = \ell n \frac{L_1}{L_1^o} + \ell n \frac{L_2}{L_2^o} + \ell n \frac{L_3}{L_3^o} = \text{volumetric strain i.e. } \ell n \left(\frac{V}{V_o} \right) = \varepsilon_a$$

$$\ell n \frac{L_1}{L_1^o} = \ell n \frac{L_1^0 + \delta L}{L_1^o} = \ell n \left(1 + \frac{\delta L}{L_1^o} \right) = \ell n (1 + \varepsilon_1) = \varepsilon_1^* \text{; at small strains } \varepsilon_1 \approx \varepsilon_1^*$$

where ε_1 is called engineering strain and ε_1^* is called the true strain. For small deformation both are equal to one another.

$$\varepsilon_a = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

For incompressible flow volumetric strain = 0.

In uniaxial deformation $\varepsilon_2 = \varepsilon_3$

For deformation at constant volume:

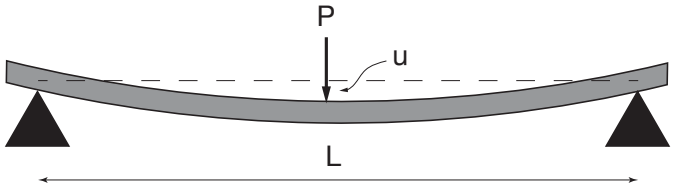
$$\varepsilon_1 + 2\varepsilon_2 = 0$$

$$\nu = -\frac{\varepsilon_2}{\varepsilon_1}$$

therefore. the constant volume condition translates into $\nu = 0.5$.

Note that a simple tensile test in an Instron measures E and ν which can then give the values for the Bulk Modulus and the Shear Modulus by using the equations given above.

Elastic constants are used in the analysis of elastic deformation of engineering structures, such as:



$$u = \frac{PL^3}{48EI} \qquad I = \frac{wt^3}{12} \qquad u = P\frac{L^3}{4Ewt^3}$$

Note that the deformation depends only on one elastic constant, E. Can you explain?