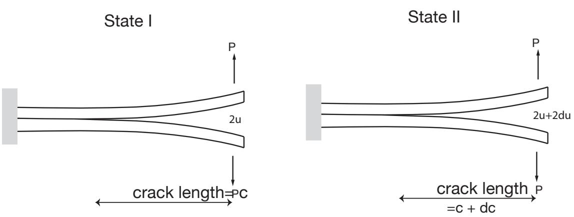
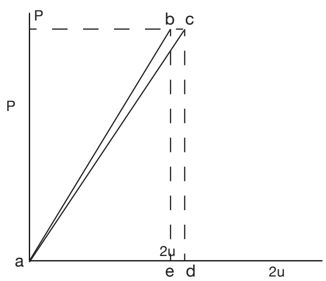
#17: Fracture: Penny shaped crack in a large body that is arbitrary loaded on its surface

Discussion of the HW-1 practice problem set.



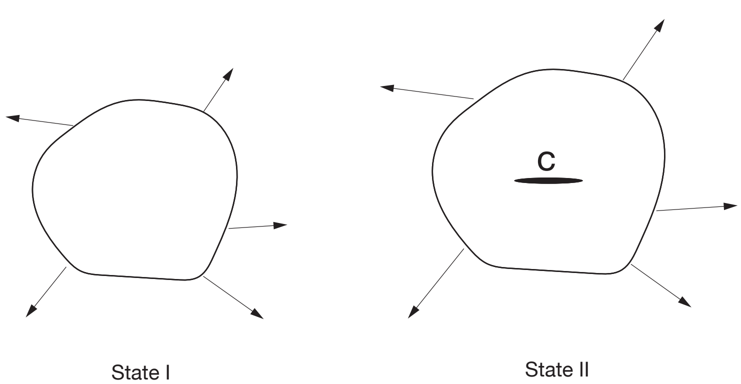


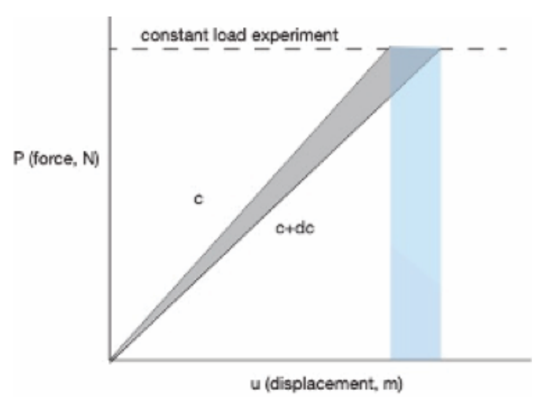
Overview of penny shaped crack in a large body

•The principal of the relationship between **the change in the potential energy** and the change in the stored elastic strain energy within the body due to the presence of a crack.

•Free energy approach to find the unstable position when the crack will propagate in an unstable manner.

Change in Potential Energy and Stored Elastic Energy (under conditions of constant load)





We shall use the Gibbs Energy approach to obtain the final solution to the problem. In this method the total energy difference (rather than the incremental change) is expressed as a change in the Gibbs free energy. Next the change in Gibbs free energy with the crack size is considered. The maximum in this curve gives the point for the onset of fracture.

H = the reduction in the potential energy of the system (it is a negative quantity)

U= increase in the stores elastic energy

$$\frac{1}{2}|H| = |U|$$

H is positive and U is negative.

Therefore the total change in mechanical work in going from State I to State II = $-|U| \; or \; -\frac{1}{2}|H|$. Therefore just knowing one of these quantities gives the total mechanical work that is available.

Let us apply above to a situation where we not know the change in the potential energy but we can calculate from mechanics the change in the stored elastic energy. Then I can assert that the total decrease in the mechanical potential will be equal to the increase in the stored elastic energy.

For a crack the increase in the surrounding elastic energy is given by

(energy per unit volume under the stress existing remotely from the crack) x (twice the effective volume of the crack)

The first quantity is

$$\frac{\sigma^2}{2E} = \int_0^\sigma \sigma \, d\varepsilon = \int_0^\sigma \sigma \frac{d\sigma}{E}$$

The second quantity is

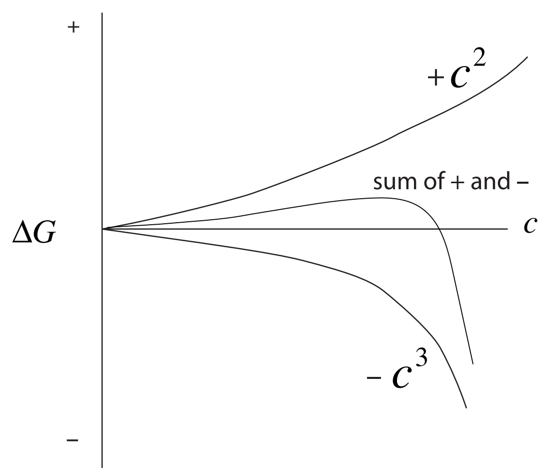
$2*\frac{4\pi}{3}\left(\frac{c}{2}\right)^3$ the effective volume is the volume of a sphere which just enclosed the crack within it.

Total change in the Gibbs Free Energy

$$\Delta G = +2\gamma_F\left(\frac{\pi c^2}{4}\right) - 2*\frac{4\pi}{3}\left(\frac{c}{2}\right)^3\frac{\sigma^2}{2E} \qquad (1)$$

A plot of this equation shows a maximum with the crack size.

The curve will have a maximum. At that maximum the crack is critical.



The criterion for fracture

$$\frac{d\Delta G}{dc} = 0$$

$$\frac{d\Delta G}{dc} = \gamma_F \pi c - \pi c^2 \frac{\sigma^2}{2E} = 0$$

$$2\gamma_F = \frac{\sigma^2 c}{E} \qquad (2)$$

Now we write

$$K_{IC}^2 = \sigma^2 c \qquad (3)^{**}$$

$$2\gamma_F = \frac{K_{IC}^2}{E} \qquad (4)^{**}$$

Notes:

•All three quantities in Eq. (4) are material parameters.

•The ability to specify fracture which propagates from a flaw in terms of a material parameter is a very significant development in fracture mechanics.

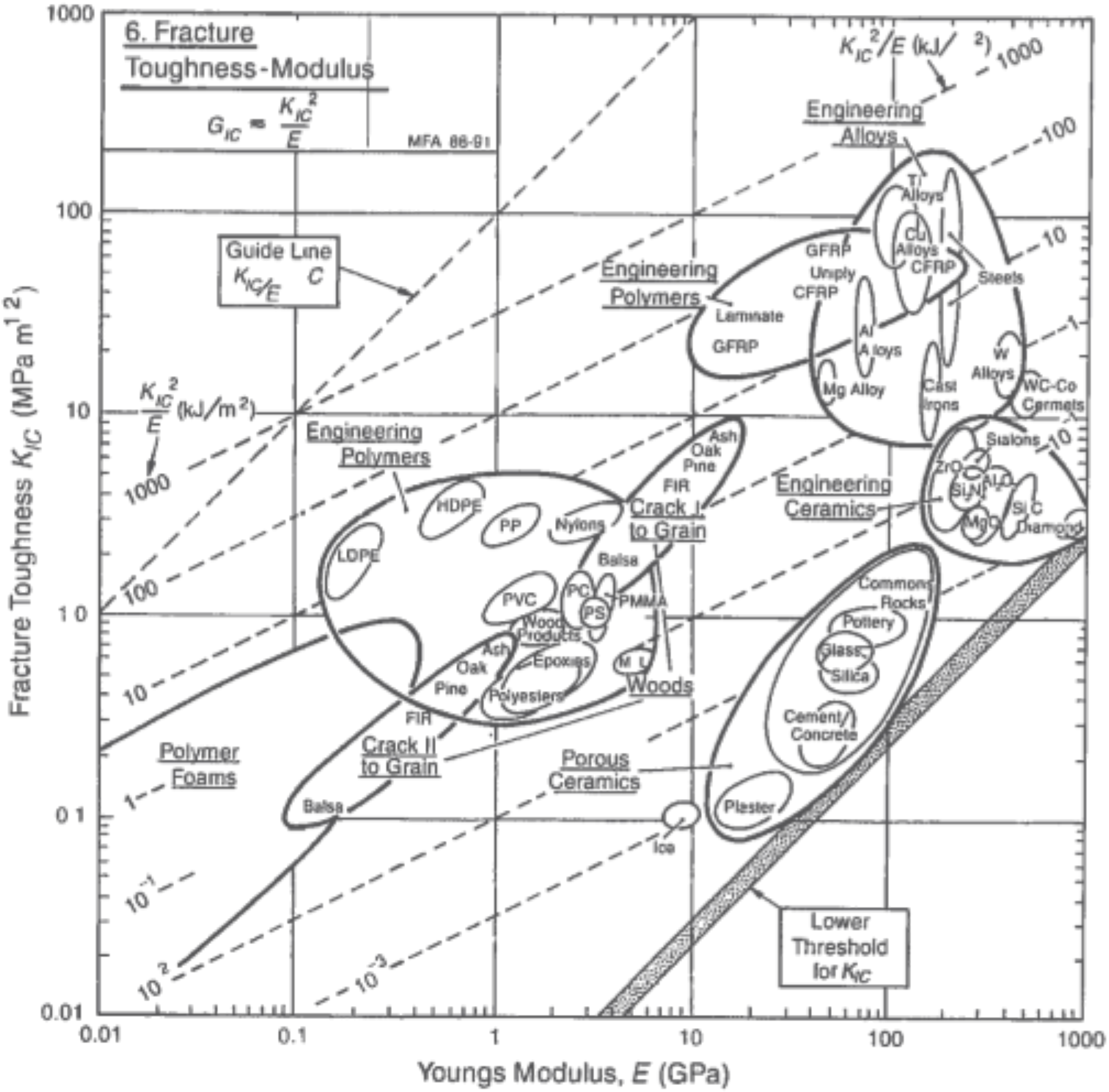


Fig. 4.8 Chart 6: Fracture toughness, K_{Ic}, plotted against Young's modulus, E. The family of lines are of constant K_{Ic}^2/E (approximately G_{Ic}, the fracture energy). These, and the guide line of constant K_{Ic}/E, help in design against fracture. The shaded band shows the 'necessary condition' for fracture. Fracture can, in fact, occur below this limit under conditions of corrosion, or cyclic loading.