Bridging Length Scales

#19: Fracture: Crack-Tip Opening Displacement

Overview

Fracture criterion is the damage at the crack tip reaching a critical level which is characterized by a critical value of the Crack Tip Opening Displacement (CTOD)

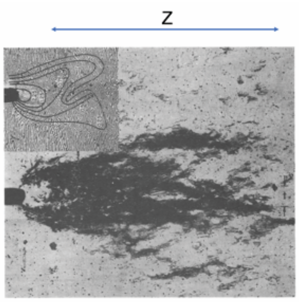


Fig. 7. Interference pattern with isostrain contours (top left corner) and the corresponding plastic zone revealed by etching, both for Sample S-56 (t = 0.017 in., T/Y = 0.81). ×17.5.

$2\gamma_F = CTOD * Y$

$2\gamma_F = \frac{K_{IC}^2}{E} = CTOD * Y$

$CTOD = \frac{K_{IC}^2}{YE}$

On the Left: Damage (pulling apart of the matrix by voids) in a specimen of tool-steel - the critical value of the damage, that is the point of fracture, is described by critical value of CTOD.

On the Right: Straightening out of fibrils in a polymer at the crack tip. They get pulled out of the matrix and stretched until they fracture. Again the fracture condition is a critical value of CTOD.

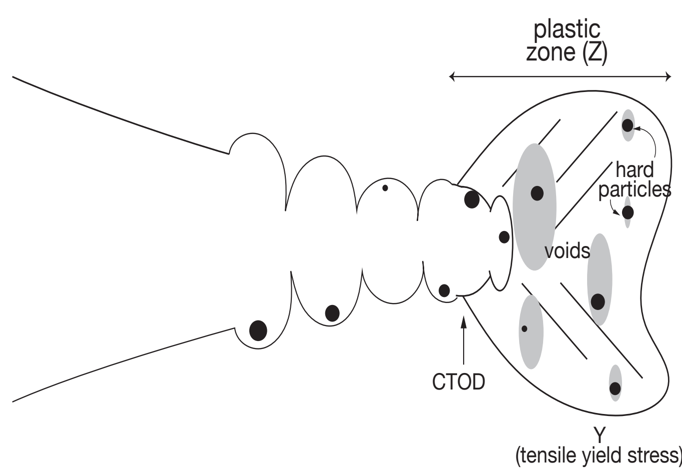
NOTE: The question arises the global fracture process, which was analyzed in terms of the change in the global elastic energy in the system, is compatible with plastic deformation at the crack tip? The answer comes in two parts: (i) The local deformation describes the mechanism for the propagation of the crack. The work done for propagating the crack can still be described in terms of the local work done per unit area of the crack. (ii) the elastic energy consideration will still apply as long that the overall energy in the system depends only the size of the crack, not in terms of what is happening near the crack tip: this condition will hold as long as the size of the plastic zone locally at the crack tip is much smaller than the overall volume of the system deforms elastically.

The Physical Parameters to Describe Damage and Fracture at the Crack Tip

Note the elements of the fracture mechanism at the crack tip. They are,

- The local stress is equal to the yield stress of the metal
- The plastic zone size is the physical distance of the damage zone in front of the crack tip
- The crack tip opening displacement (CTOD) is the local "tearing apart" that is when the damage causes separation and the crack moves forward.

We must now consider the above parameters and build them into a model that related them to the measured value of the fracture toughness.



The analysis for the Work-of-Fracture in terms of the tensile yield stress, the damage zone and CTOD

The work done per unit area growth of the crack is a product of the local stress, which is equal to the tensile yield stress times the "critical value" of the crack tip opening displacement.

$$2\gamma_F = \sigma_Y CTOD \tag{1}$$

Note how the area of the crack cancels out on both sides. Let us say that the crack extends by an area ΔA. Multiplying both sides by ΔA converts the left hand side to units of energy (rather than energy per unit area), and on the right hand side σ_Y into force, which multiplied by CTOD gives energy.

Now we wish to relate Eq. (1) to quantities to engineering parameters that is, K_{IC}. We therefore use the global equation

$$2\gamma_F = \frac{K_{IC}^2}{E} \tag{2}$$

Combining the Eqns (1) and (2),

$$CTOD = \frac{K_{IC}^2}{\sigma_Y E} \tag{3}$$

Let us apply to aluminum alloys. The engineering parameters are obtained from the maps (K_{IC} vs. E, and yield stress vs. E)

Aluminum Alloys

E	80GPa	80000MPa
K_{IC}	40MPa $m^{0.5}$	
Yield Stress	500MPa	
CTOD	0.00004m	
	0.04mm	
	40µm	

Plastic Zone Size (Z)

Question is how are CTOD and Z related to one another?

$$\text{The local plastic strain} = \frac{CTOD}{Z} \tag{4}$$

Concept: the local plastic strain is equal to the elastic strain in the region surrounding the plastic zone.

$$\text{The elastic strain} = \frac{\sigma_Y}{E} \tag{5}$$

$$Z = \frac{K_{IC}^2}{\sigma_Y^2} \tag{6}$$

Yield Stress	500 MPa
K_{IC}	40 MPa $m^{0.5}$
Z	0.0064 m
	6.4 mm