Bridging Length Scales

24: Strain rate Equation for Steady State Creep

Flow Chart

•Describe the geometrical relationship between the movement of atoms and strain (Lect 22)

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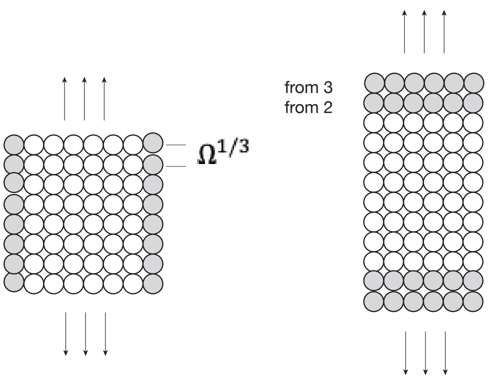

•Consider the rate of movement of atoms by diffusion (Lect 23)

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•Describe the equations the prescribe the influence of applied stress to the diffusion flux (Lect 24)

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Where we wish to arrive:

$$\dot{\varepsilon} = A\frac{\sigma^n}{d^p}D_O e^{-\frac{Q}{RT}}$$

$$\log\dot{\varepsilon} = \log A + n\log\sigma - p\log d + \log D_O - \frac{(Q/1000)}{2.3*1000R}\frac{1000}{T} \qquad \textbf{(1)}$$

σ is the uniaxial stress

d is the grain size

D_O is the pre-exponential for the diffusion coefficient

Q is the activation energy for overcoming the energy barrier for atom jumps.

Separation of Variables in Eq. (1)

Note how the variables in Eq. (1) are separated into different entities, that is, the grain size dependence does not influence temperature dependence, or the stress dependence does not influence grain size dependence etc.

For example, consider the data on superplastic deformation of zirconia we discussed at the start of the topic of Deformation at High Temperature. It shows how the strain rate varies with stress at different temperatures. Let us consider how we can obtain values for the stress exponent, n, and the activation energy Q_V from these data, applying equation

$$\dot{\varepsilon} = A\frac{\sigma^n}{d^p}D_O e^{-\frac{Q}{RT}}$$

$$\log\dot{\varepsilon} = \log A + n\log\sigma - p\log d + \log D_O - \frac{(Q/1000)}{2.3*R}\frac{1000}{T}$$

•For example to obtain the stress exponent consider the slope of the lines (for a given temperature) on this log-log plot. In the $\log_{10}$ scale a factor of ten becomes one unit length. As drawn in the pink lines the slope is a bit less than n=2 since the slope in the figure shows that the strain rate changes by two orders of magnitude for one magnitude change in the stress. The data have a slightly lower slope than n=2, but it is greater than n=1, so $1 \le n \le 2$.

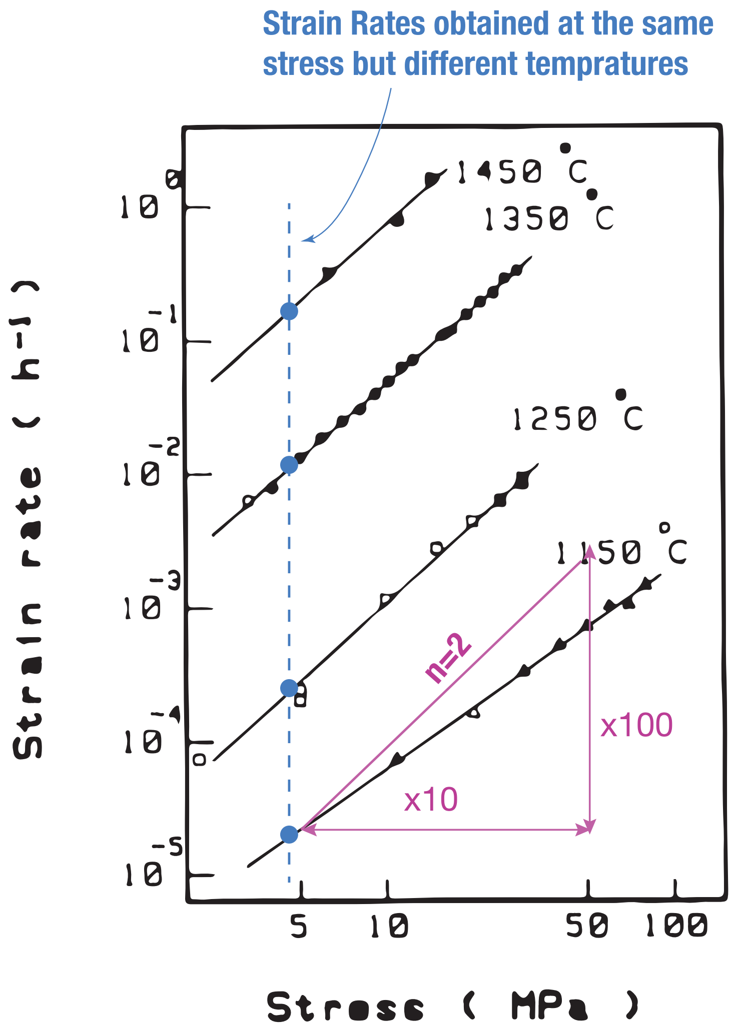
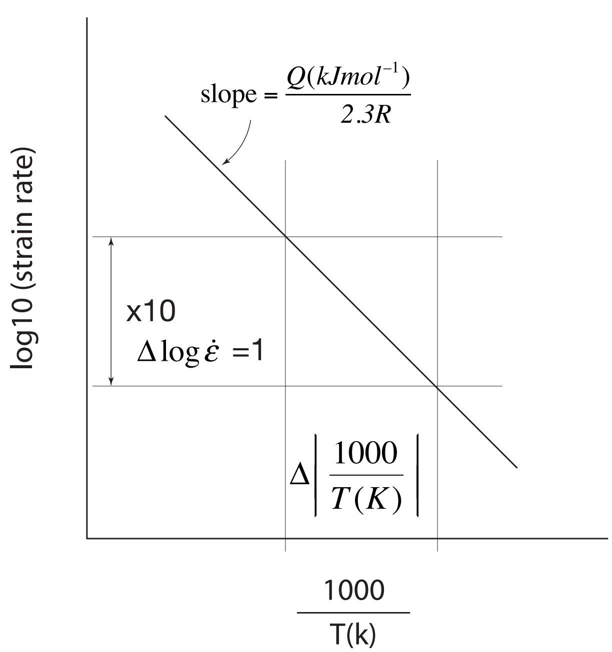


Fig. 3. Steady-state creep rate vs applied stress for Y-TZP

Let us now consider how to obtain a value for Q_V from the temperature dependence of the strain rate at a constant stress as shown in "blue".

The data are plotted in an "Arrhenius" plot as shown below where the logarithm of strain rate is plotted against (1/T), which as prescribed by the expanded equation written just above would result in a straight line such that the slope will yield the value of Q_V. The procedure is laid out graphically in the schematic given just below,

$$Slope = \frac{\Delta\log\dot{\varepsilon}}{\Delta\left[\frac{1000}{T(K)}\right]} \qquad Q(kJ\,mol^{-1}) = 2.3R\frac{(\Delta\log\dot{\varepsilon})}{\Delta\frac{1000}{T}}$$



Now let us derive the equation for the strain rate

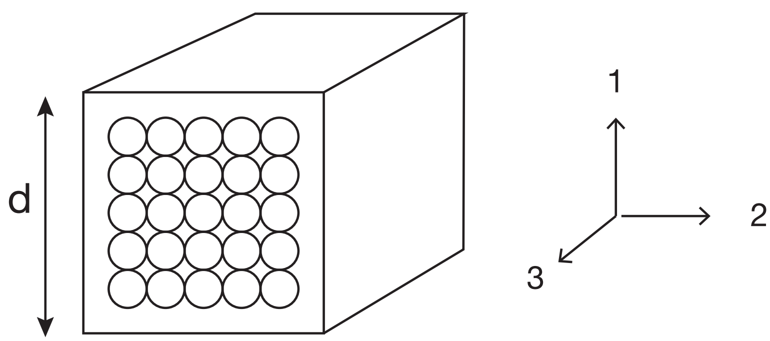
We are seeking to derive an equation of the following form

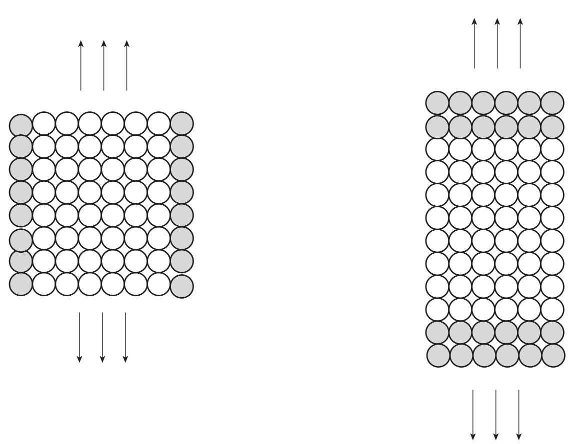
$$\dot{\varepsilon} = A\frac{\sigma^n}{d^p}D_O e^{-\frac{Q}{RT}}$$

where are paramters were described at the start of the lecture. The mechanism that we wish to invoke is given described in the figure given just below

We start with the equation for the diffusional flux (in units of atoms flowing m^{-2} s^{-1}.

$$J = \frac{D_V}{\Omega k_B T}\frac{2\sigma\Omega}{d} \quad \text{m}^2\text{s}^{-1} \qquad (*)$$





note that the gradient of the chemical potential which drives the atoms from the side faces to the "normal" faces is given by $\frac{2\sigma\Omega}{d}$, because the diffusion distance which gives the gradient is (d/2)

Steps to construct a strain rate equation from above are described starting on the following page.

J	J flux of atoms flowing from side to the normal face in units of #atoms $m^{-2}s^{-1}$
ϕ	ϕ The total number of atoms being transported from the side to the vertical faces= $J * d * (d/2) * 4$
$\dot{V}$	$\dot{V}$ is the volume of atom flowing into the vertical faces= $\phi\Omega$
$\dot{d}$	$\dot{d}$ is the thickening of the grain size in the vertical direction $\dot{d} = \frac{\dot{V}}{d^2}$
$\dot{\varepsilon}$	$\dot{\varepsilon}$ is the strain being imposed on the grain $\dot{\varepsilon} = \frac{\delta d}{dt}\frac{1}{d}$

$$\dot{\varepsilon} = \frac{1}{d}\frac{1}{d^2}\Omega\frac{4d^2}{2}\frac{D_V}{\Omega k_B T}\frac{2\sigma\Omega}{d}$$

$$\dot{\varepsilon} = \frac{1}{d}\frac{4}{}\frac{D_V}{k_B T}\frac{\sigma\Omega}{d} = 4\frac{\sigma\Omega}{k_B T}\frac{D_V}{d^2}$$

$$\dot{\varepsilon} = 4\frac{\sigma\Omega}{k_B T}\frac{D_V}{d^2}$$

$$\dot{\varepsilon} = A\frac{\sigma^n}{d^p}D_O e^{-\frac{Q}{RT}}$$

n=1

D_V is the diffusion coefficient

d^2 for the grain size dependence.

$$\dot{\varepsilon} = 4\frac{\sigma\Omega}{k_B T}\frac{D_V}{d^2}$$

$$\sigma = \left[\frac{k_B T d^2}{4\Omega D_V}\right]\dot{\varepsilon}$$

$$\sigma = \eta\dot{\varepsilon}$$

in fluids the viscosity is related to the diffusion coefficient

$$\eta = \frac{k_B T}{6\pi\Omega^{1/3}D} \qquad \sigma_s = \eta\dot{\varepsilon}$$

The lower bound for the grain size is one atom which has a size of $d = \Omega^{1/3}$; note how this derivation recovers the Stokes Einstein Equation that relates viscosity to the diffusion coefficient which we discussed at the start of the lecture on Diffusion.